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ADIABATIC COMPRESSION OF A GAS BY MEANS OF A SPHERICAL DRIVER

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UDC 533.21

§1. A spherical driver* with an initial radius r_0 within which there is a gas at rest ($\varkappa$ is the polytropic index; c_0 is the velocity of sound) starts to converge toward the center at a certain time. The problem is the determination of that driver trajectory for which all β characteristic curves emerging from it converge at the center of the time of collapse of the driver, which is taken to be the origin of the time scale, t = 0, in the following. In this case the motion of the gas within the driver will be spherically symmetric, isentropic, and self-similar. We take $\eta = c_0 t/r$ as the self-similar variable, and the gasdynamical functions are represented in the form

$$u = r/tu_1(\eta);\ \ c = r/tc_1(\eta).$$

In the r–t plane, the flow will be separated from the region at rest by the characteristic curve $r = -c_0 t$ ($\eta = -1$). The functions $u_1(\eta)$ and $c_1(\eta)$ are defined by the equation

$$\frac{du_1}{dc_1^2} = \frac{u_1\left[(u_1-1)^2 - 3c_1^2\right]}{2c_1^2\left[(u_1-1)(\varkappa u_1-1) - c_1^2\right]} \tag{1.1}$$

*A solution is given in [1] for the case of a plane driver. A self-similar spherically symmetric compression wave was also considered by I. E. Zababakhin and V. A. Simonenko. (Private communication — Ya. K.).

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 23-30, January-February, 1977. Original article submitted February 26, 1976.

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with the initial conditions

$$u_1 = 0, \quad c_1^2 = 1.$$

and by the quadrature

$$\eta = -\exp \int_1^{c_1^2} \frac{(u_1-1)^2 - c_1^2}{2c_1^2\left[(u_1-1)(\varkappa u_1 - 1) - c_1^2\right]}\, dc_1^2. \tag{1.2}$$

The point M ($u_1 = 0$, $c_1^2 = 1$) is a singular point of Eq. (1.1). The nature of the singularity is a node. The separating strand of the node is $u_1 = 0$. The asymptote corresponding to the general direction is

$$c_1^2 \approx 1 + (\varkappa + 1) u_1 \ln u_1 + A u_1,$$

where A is an arbitrary constant.

The integral curves of Eq. (1.1) which intersect the curve L [$(u_1 - 1)^2 = c_1^2$] outside singular points correspond to flow in which shock waves arise, i.e., the characteristic curves intersect before collapse of the driver. In fact, according to the quadrature (1.2), u_1 and c_1^2 cease to be single-valued functions of η in this case. It is clear from the diagram shown for the isoclines for Eq. (1.1) that only the integral curve connecting the singular points M and N [$u_{10} = 2/(3\varkappa - 1)$, $c_{10}^2 = 3(\varkappa - 1)^2/(3\varkappa - 1)^2$] does not intersect the curve L [analysis of Eq. (1.1) indicates that no integral curve passes through the point $u_1 = \pm\infty$, $c_1^2 = \infty$] (Fig. 1).

The singular point N is a saddle point. It is therefore appropriate to make a numerical determination of the integral curve MN between the points M and N. To do this, we make the following substitution of variables in Eq. (1.1): $x = c_1^2 - c_{10}^2$, $y = u_1 - u_{10}$. To the separatrix of the saddle N going to the point M there corresponds the expression

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{2(1-3\varkappa)}{(\varkappa - 1)\left[3 - \varkappa + \sqrt{13\varkappa^2 - 14\varkappa + 5}\right]}.$$

According to the quadrature (1.2), the value $\eta = 0$ corresponds to the point N. Since the integrand in the quadrature (1.2) goes neither to zero nor infinity in the interval $1 > c_1^2 > 3(\varkappa - 1)^2/(3\varkappa - 1)^2$, η increases from -1 to 0. The curve $u_1(\eta)$, $c_1(\eta)$ corresponding to the integral curve of Eq. (1.1) that connects the points M and N will be the desired curve. In fact, the β characteristic curves are defined by the equation

$$\frac{dt}{d\eta} = \frac{t}{\eta\left[1 + c_1 - u_1\right]}. \tag{1.3}$$

Since the integral curve MN outside the point M corresponding to $\eta = -1$ does not intersect the curve L, the denominator on the right side of Eq. (1.3) is different from zero in the interval $-1 < \eta < 0$ and the entire right side is positive for $t < 0$. Thus, η is a monotonically increasing function of t and a single characteristic curve passes through each point ($t < 0$, $\eta < 0$).

The asymptote to any β characteristic curve when $\eta \to 0$ has the form

$$(-t) \sim (-\eta)^{(3\varkappa-1)/[(3-\sqrt{3})(\varkappa-1)]} \tag{1.4}$$

or

$$r \approx K(-t)^{\alpha}, \text{ where } \alpha = [2 + \sqrt{3}(\varkappa - 1)]/(3\varkappa - 1),$$

and the asymptote to any trajectory, in particular the driver trajectory, when $\eta \to 0$ is

$$(-t) \sim (-\eta)^{(3\varkappa-1)/3(\varkappa-1)} \text{ or } \quad r \approx G(-t)^{2/(3\varkappa-1)}$$

Here K and G are positive constants corresponding to a given characteristic curve or a given trajectory. The trajectory of the driver can be determined numerically by integration of the equation

$$t\,d\eta/dt = \eta(1 - u_1), \; t = -r_0/c_0, \; \eta = -1.$$

In the neighborhood of the driver, the asymptotes of the gasdynamical functions appear to be the following: density, $\sim r^{-3}$; velocity and sound velocity, $\sim r^{3(1-\varkappa)/2}$; pressure, $\sim r^{-3\varkappa}$; temperature, $\sim r^{3(1-\varkappa)}$. These asymptotic values are also valid for other modes of adiabatic compression of a gas [2-4].

§2. An abrupt deceleration of the driver at some time $t_1 (t_0 < t_1 < 0)$ entails a disturbance of the resultant solution for a centered rarefaction wave with a peak at the point r_1, t_1 (r_1 is the distance of the driver from the center at the time t_1). The limit of the disturbance is the β characteristic curve of the original solution emerging from the point (r_1, t_1) if no β characteristic curve of the disturbed flow intersects it in the interval $t_1 < t < 0$. For this, it is necessary that

$$u = u_0(t) + u_1(t)(g-1) + u_2(t)(g-1)^2 + \ldots,$$
$$c^2 = f = f_0(t) + f_1(t)(g-1) + f_2(t)(g-1)^2 + \ldots, \qquad (2.1)$$

as the asymptotic representations of the functions in its neighborhood, where $g = r/\varphi(t)$; $r = \varphi(t)$ is the equation for this separating β characteristic curve; the coefficients $u_i(t)$ and $f_i(t)$ are finite in the interval $t_1 < t < 0$ and $u_i(t)/u_0(t) \sim O(1)$ for $t \to 0$. It is obvious that $u_0(t)$ and $f_0(t)$ are values of the corresponding functions on the β characteristic curve $r = \varphi(t)$ obtained in the original solution. The equations defining them and the function $\varphi(t)$ and the initial data have the form

$$\frac{du_0}{dt} = \frac{2u_0 f_0 t}{[(u_0 + c_0)t - \varphi]\varphi};$$
$$\frac{df_0}{dt} = -\frac{2(\varkappa - 1)f_0(u_0 t - \varphi)}{[(u_0 + c_0)t - \varphi]\varphi}; \quad \frac{d\varphi}{dt} = u_0 - c_0; \quad c_0 = \sqrt{f_0};$$
$$t = t_1: \quad u_0 = r_1/t_1 u_1(c_0 t_1/r_1); \quad f_0 = r_1^2/t_1^2 c_1^2(c_0 t_1/r_1); \quad \varphi = r_1.$$

The functions $f_1(t)$ and $u_1(t)$ are defined by a single differential equation

$$2\varphi df_1/dt + f_1[2(\varkappa - 2)u_0 c_0 - (\varkappa + 2)\varphi du_0/dt - \varphi dc_0/dt]/c_0 -$$
$$- (\varkappa + 1)f_1^2/(\varkappa - 1)c_0 + (\varkappa - 1)\{\varphi^2[c_0 d^2u_0/dt^2 - (du_0/dt_0)^2] -$$
$$- \varphi du_0/dt(\varphi dc_0/dt + 2f_0)\}/c_0 - 2(\varkappa - 1)u_0 f_0 = 0 \qquad (2.2)$$

and the final relation

$$u_1 = -[f_1/(\varkappa - 1) + \varphi du_0/dt]/c_0. \qquad (2.3)$$

Initial data for the function $f_1(t)$ can be obtained from consideration of the rarefaction wave at times close to the time t_1. Asymptotic values of the functions u and f for small values of $t - t_1$ are represented in the form [5]

$$u = U_0(\xi) + U_1(\xi)(t - t_1) + U_2(\xi)(t - t_1)^2 + \ldots,$$
$$f = F_0(\xi) + F_1(\xi)(t - t_1) + F_2(\xi)(t - t_1)^2 + \ldots, \qquad (2.4)$$
$$\xi = (r - r_1)/(t - t_1).$$

It is evident that

$$\xi \geqslant \xi_0 = \varphi'(t_1) = u_0(t_1) - c_0(t_1).$$

The functions $U_0(\xi)$ and $F_0(\xi)$ are known self-similar solutions for a plane rarefaction wave

$$U_0(\xi) = [(h-1)/h]\xi + A/h, \; F_0(\xi) = (\xi - A)^2/h^2,$$

where

$$h = (\varkappa + 1)/(\varkappa - 1); \; A = u_0(t_1) + (h - 1)c_0(t_1).$$

Expressions can also be obtained for the functions $U_1(\xi)$ and $F_1(\xi)$. In particular,

$$U_1(\xi) = -\frac{1}{r_1}\left[\frac{2}{(h-1)(2-h)}\frac{(\xi - A)}{h} + \frac{3(h-1)}{4-h}\left(\frac{\xi - A}{h}\right)^2 + B\left(\frac{\xi - A}{h}\right)^{h/2}\right],$$
$$F_1(\xi) = (\varkappa - 1)\left[\frac{2-3h}{h+2}\left(\frac{\xi - A}{h}\right)U_1(\xi) - \frac{2h}{(h+2)r_1}\frac{(h-1)\xi + A}{h}\left(\frac{\xi - A}{h}\right)^2\right]. \qquad (2.5)$$

The constant B is determined by matching the asymptotic values (2.4) with the values of the functions on the characteristic curve $r = \varphi(t)$. In the cases $h = 2$ and $h = 4$, the terms in Eq. (2.5) having coefficients going to infinity are respectively replaced by $\ln[(\xi - A)/h]$ and $(\xi - A)/h \ln[(\xi - A)/h]$ with finite coefficients. The definitions of g and ξ indicate that the asymptotic relation

$$\xi - \xi_0 \approx [\varphi(t_1)/(t - t_1) + \varphi'(t_1)](g - 1)$$

is valid for small values of $t - t_1$, $\xi - \xi_0$, and $g - 1$, and we have from Eq. (2.4) the asymptotic value

$$f = F_0(\xi_0) + F_1(\xi_0)(t - t_1) + [F_0'(\xi_0) + F_1'(\xi_0)(t - t_1)] \times [\varphi(t_1)/(t - t_1) + \varphi'(t_1)](g - 1).$$

Consequently, for small values of $t - t_1$,

$$f_1(t) \approx [F_0'(\xi_0) + F_1'(\xi_0)(t - t_1)][\varphi(t)/(t - t_1) + \varphi'(t_1)].$$

Since $F_0'(\xi_0) \neq 0$, and $\varphi'(t_1)$ and $F_1'(\xi_0)$ are finite, in the leading term

$$f_1(t) \approx F_0'(\xi_0)\varphi(t_1)/(t - t_1). \tag{2.6}$$

Thus it is necessary to determine the function $f_1(t)$ satisfying Eq. (2.2) and having the asymptotic value (2.6) for $t \to t_1$. Equation (2.2) is a Riccati equation. Its particular solution corresponding to the expansion (2.1) in the undisturbed region is well known:

$$\Phi(t) = \frac{-4u_0 f_0(u_0 t - \varphi)t}{(h-1)(\varphi' t - \varphi)[(u_0 + c_0)t - \varphi]}.$$

Through the substitution

$$z = 1/(f - \Phi)$$

Eq. (2.3) is reduced to the linear equation

$$dz/dt - L(t)z + D(t) = 0,$$

where

$$D(t) = (\varkappa + 1)/[2(\varkappa - 1)\varphi c_0];$$

$$L(t) = \frac{2(\varkappa+1)}{\varphi}\frac{u_0 c_0(u_0 t - \varphi)t}{(\varphi' t - \varphi)[(u_0 + c_0)t - \varphi]} + \frac{2(\varkappa - 2)u_0 c_0 - (\varkappa + 2)\varphi u_0' - c_0'\varphi}{2\varphi c_0}.$$

The particular solution $\Phi(t)$ is finite in the interval $t_0 < t < 0$, since the selected integral curve of Eq. (1.1) does not intersect the curve L $[(u_1 - 1)^2 = c_1^2]$. Consideration of Eq. (2.6) indicates that when $t \to t_1$,

$$z \approx \frac{t - t_1}{\varphi(t_1)F_0'(\xi_0) - \Phi(t_1)} \to 0$$

and, consequently,

$$z(t) = \int_{t_1}^{t} D(t)\exp\left[-\int_{t_1}^{t} L(\tau)\,d\tau\right]dt \exp\left[\int_{t_1}^{t} L(\tau)\,d\tau\right].$$

Since $L(t)$ is finite and $D(t)$ conserves its sign in the interval $t_1 < t < 0$, $z(t) \neq 0$ in that interval and the function $f_1(t) = \Phi(t) + 1/z(t)$ is finite because of the finiteness of $\Phi(t)$. The finiteness of $u_1(t)$ follows from Eq. (2.3) because the function $c_0(t) \neq 0$ in the half-open interval $t_1 \leq t < 0$. The functions f_n and u_n are determined from the linear equation and final relation

$$\begin{gathered}\{[\varphi df_n/dt + u_n f_1 + 2u_{n-1}f_2 + n(u_1 - \varphi')f_n]/(\varkappa - 1) + 2f_0[(-1)^n u_0 + \\ + (-1)^{n-1}u_1 + \ldots + u_n] + \ldots + f_n(u_1 + 2u_0)\} - \sqrt{f_0}\{\varphi du_n/dt + u_n u_1 + 2u_{n-1}u_2 + \ldots + n(u_1 - \varphi')u_n\} = 0, \\ f_n = -(\varkappa - 1)\{\sqrt{f_0}u_n + [\varphi u_{n-1}' + u_{n-1}u_1 + 2u_{n-2}u_2 + \ldots + (n-1)(u_1 - \varphi')u_{n-1}]/n\}.\end{gathered} \tag{2.7}$$

On the half-open interval $t_1 \leq t < 0$, the coefficient of the derivative is different from zero and the remaining terms are finite; consequently, $u_n(t)$ and $f_n(t)$ are finite on this half-open interval.

§3. The results obtained provide an opportunity to determine an asymptotic value for the disturbed flow in the neighborhood of the center at $t \to 0$ near the separating β characteristic curve $r = \varphi(t)$. Since the singular point N $[u_{10} = 2/(3\varkappa - 1),\ c_{10}^2 = 3(\varkappa - 1)^2/(3\varkappa - 1)^2]$ of Eq. (1.1) corresponds to $\eta = 0$, then according to Eq. (1.4)

$$u_0(t) = \varphi(t)/tu_1(\eta)_{\eta\to\infty} \approx -2/(3\varkappa - 1)K(-t)^{\alpha-1},$$

$$f_0(t) = (\varphi(t)/t)^2 f_1(\eta)_{\eta\to 0} \approx 3(\varkappa - 1)^2/(3\varkappa - 1)^2 K^2(-t)^{2(\alpha-1)},$$

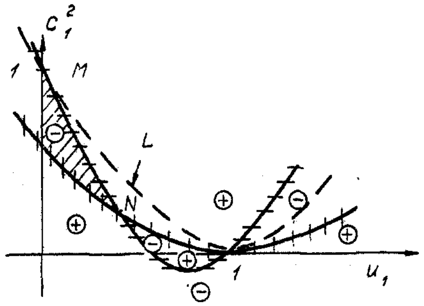

Fig. 1

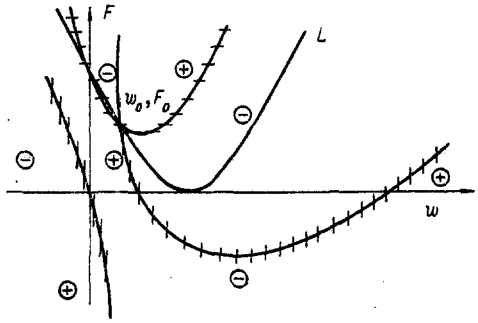

Fig. 2

because of which we have from Eq. (2.3)

$$u_1(t) \approx \frac{(3-\sqrt{3})(5\varkappa-3)}{\sqrt{3}(3\varkappa-1)(\varkappa+1)} K(-t)^{\alpha-1},$$

$$f_1(t) \approx \frac{(\sqrt{3}-3)(3\varkappa-5)(\varkappa-1)^2}{(3\varkappa-1)^2(\varkappa+1)} K^2(-t)^{2(\alpha-1)}.$$

That $u_n(t)$ and $f_n(t)$ are of the same order of magnitude follows from Eq. (2.7); substitution of the expressions obtained in Eq. (2.1) yields the asymptotic value for $t \to 0$ near the separating β characteristic curve $r = \varphi(t)$,

$$u \approx -K(-t)^{\alpha-1}\left[\frac{2}{3\varkappa-1} - \frac{(3-\sqrt{3})(5\varkappa-3)}{\sqrt{3}(3\varkappa-1)(\varkappa+1)}\left(\frac{r}{K(-t)^{\alpha}}-1\right)+\ldots\right],$$

$$f \approx K^2(-t)^{2(\alpha-1)}\left[\frac{3(\varkappa-1)^2}{(3\varkappa-1)^2} + \frac{(\sqrt{3}-3)(\varkappa-1)^2(3\varkappa-5)}{(\varkappa+1)(3\varkappa-1)^2}\left(\frac{r}{K(-t)^{\alpha}}-1\right)+\ldots\right]. \quad (3.1)$$

Equations (3.1) provide a basis for seeking the asymptotic value in the neighborhood of the center when $t \to 0$ in the form

$$u = \pm K|t|^{\alpha-1}\zeta w(\zeta),\ f = K^2|t|^{2(\alpha-1)}\zeta^2 F(\zeta),\ \zeta = r|t|^{-\alpha}/K(+ \text{ for } t>0, - \text{ for } t<0). \quad (3.2)$$

The functions w and F are defined by the equation

$$\frac{dF}{dw} = \frac{(3\varkappa-1)F\{2(F-\alpha)-2\varkappa w^2+[5-\varkappa+\sqrt{3}(\varkappa-1)w]\}}{F[3(3\varkappa-1)w-2(3-\sqrt{3})]+(3\varkappa-1)w(1-w)(w-\alpha)} \quad (3.3)$$

with the initial data

$$w = w_0 = 2/(3\varkappa-1);\ F = F_0 = 3(\varkappa-1)^2/(3\varkappa-1)^2$$

and by the quadrature

$$\zeta = \exp\int_{w_0}^{w} \frac{[(w-\alpha)^2-F](3\varkappa-1)\,dw}{F[3(3\varkappa-1)w-2(3-\sqrt{3})]+(3\varkappa-1)w(1-w)(w-\alpha)}. \quad (3.4)$$

The initial point (w_0, F_0) is a node for Eq. (3.3). This is clear from the isocline diagram (Fig. 2). From the condition for matching the solution of Eq. (3.3) to the asymptotic value (3.1), it follows that emergence from the node must be along the direction of its separating strand,

$$dF/dw = \sqrt{3}(\varkappa-1)^2/(3\varkappa-1) \quad (3.5)$$

in order to provide an increase in ζ in the direction of decreasing w. Since F = 0 is an integral of Eq. (3.3), the selected integral curve unavoidably reaches the origin w = 0, F = 0 (see Fig. 2). This point is a dicritical node and in its neighborhood

$$F \approx Aw^2\ (A > 0 - \text{arbitrary constant}). \quad (3.6)$$

The quadrature (3.4) indicates that the value $\zeta = \infty$ corresponds to this point, i.e., the line t = 0, where

$$\zeta \sim w^{-\alpha}. \quad (3.7)$$

Further continuation of an integral curve of Eq. (3.3) leads to intersection with the curve L $[F = (w - \alpha)^2]$. In this case, the uniqueness of the functions $w(\zeta)$ and $F(\zeta)$ breaks down, which means the formation of a reflected shock wave.

§4. Because of self-similarity, the line $\zeta = \zeta_b$ = const corresponds to the reflected shock front, and conditions in the shock wave can be represented in the form

$$\frac{\rho_1}{\rho_0} = \frac{\alpha - w_0(\zeta_b)}{\alpha - w_1(\zeta_b)} = \frac{F_0(\zeta_b) + \varkappa[\alpha - w_0(\zeta_b)]^2}{F_1(\zeta_b) + \varkappa[\alpha - w_1(\zeta_b)]^2} = \frac{(\varkappa+1)\rho_1 F_1 + (\varkappa - 1)\rho_0 F_0}{(\varkappa - 1)\rho_1 F_1 + (\varkappa+1)\rho_0 F_0}. \quad (4.1)$$

The ratio of densities and of the squares of the sound velocities, and, consequently, of the pressures, is constant. Since the flow ahead of the shock front is isentropic, it remains that way behind the front. Thus Eq. (3.3) and the quadrature (3.4) also describe the flow behind the reflected shock front. The initial data are determined by the requirements for zero velocity and for the finiteness of the velocity of sound at the center for times $t > 0$. The quantity ζ at the center is zero for $t > 0$. Therefore, the velocity of sound at the center can be finite only under the condition $F(0) = \infty$. Then, as follows from the quadrature (3.4), the value of ζ goes to zero only for the value $w(0) = 2(3 - \sqrt{3})/3(3\varkappa - 1)$. Considering this, it is convenient to study the solution in the neighborhood of the center at times $t > 0$ in the variables

$$\chi = 1/F; \ \tau = w - w(0).$$

In that case, Eq. (3.3), the initial data, and the quadrature (3.4) take the form

$$\frac{1}{\chi}\frac{d\chi}{d\tau} = -\frac{2 + \{-2\varkappa(\tau + w(0))^2 + [5 - \varkappa + \sqrt{3}(\varkappa - 1)](\tau + w)(0) - 2\alpha)\}}{3\tau + (\tau + w(0))(1 - w(0) - \tau)(\tau + w(0) - \alpha)\chi};$$

$$\zeta = \zeta_b \exp\int\limits_{\tau(\zeta_b)}^{\tau} \frac{[\tau + w(0) - \alpha]^2\chi - 1}{3\tau + (\tau + w(0))(1 - w(0) - \tau)(\tau + w(0) - \alpha)\chi}\,d\tau. \quad (4.2)$$

The point $\chi = 0$, $\tau = 0$ is a singular point of Eq. (4.2). The singularity is a saddle point. The desired integral curve is the separatrix of the saddle with the angular coefficient of the tangent initially

$$\frac{d\chi}{d\tau} = \frac{135(3\varkappa - 1)^2}{2\sqrt{3}(3 - \sqrt{3})[9(\varkappa - 1) + 2\sqrt{3}]}.$$

The wave front and the values of the gasdynamical functions on it are determined in the following manner. Values of w_1, f_1 are determined from Eq. (4.1) for each point w_0, f_0 of the integral curve of Eq. (3.4) when $w_0 < 0$. The point of intersection of the curve $F_1 = F_1(w_1)$ obtained in this manner with the integral curve of Eq. (4.2) corresponds to the values of the gasdynamical functions on the wave front, and the corresponding value of ζ_b is determined from the quadrature (3.4). From numerical calculations performed for the value $\varkappa = 5/3$, we obtain in this way the values $\zeta_b = 6.9826$, $w_0(\zeta_b) = -0.22115$, $F_0(\zeta_b) = 0.38587$, $w_1(\zeta_b) = 0.24992$, and $F_1(\zeta_b) = 0.62893$. According to the representation (3.2) of the functions, we find that at the reflected shock front the velocity $u \sim r^{(\sqrt{3}-3)(\varkappa-1)/[2+\sqrt{3}(\varkappa-1)]}$, the density $\rho \sim r^{2(\sqrt{3}-3)/[2+\sqrt{3}(\varkappa-1)]}$; the pressure $p \sim r^{2\varkappa(\sqrt{3}-3)/[2+\sqrt{3}(\varkappa-1)]}$, and the temperature $T \sim r^{2(\varkappa-1)(\sqrt{3}-3)/[2+\sqrt{3}(\varkappa-1)]}$. From the asymptotic values (3.6) and (3.7) it follows that these quantities are of the same order of magnitude on the line $t = 0$ also and the asymptotic value (3.5) indicates that density $\sim t^{2(\sqrt{3}-3)/(3\varkappa-1)}$, pressure $\sim t^{2\varkappa(\sqrt{3}-3)/(3\varkappa-1)}$, and temperature $\sim t^{2(\varkappa-1)(\sqrt{3}-3)/(3\varkappa-1)}$ at the center for $t > 0$ and $t \to 0$.

The author is grateful to L. V. Al'tshuler for proposing this problem, to G. Kh. Solov'ev and A. E. Lutskyi for numerical computer calculations, and to N. I. Kurancheva for formulation of the work.

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